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Modelling and Forecasting Exchange Rate Dynamics: An Application of Asymmetric Volatility Models

Rohan Longmore and Wayne Robinson¹
Research Services Department
Bank of Jamaica

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Abstract

This paper compares the performance of linear GARCH models in forecasting the volatility of returns in the foreign exchange market with that of asymmetric models. We study the information content of macroeconomic and market microstructure variables for forecasting over a 30-day horizon. The paper also examines the relevance of volatility spill-overs using multivariate GARCH. A long memory process was found for the exchange rate, with the effects of shocks being asymmetric. The non-linear GARCH model did better than the linear models in terms of the explanatory power. These models also performed well in the out-of-sample forecasts, although the model that accounted for excessive kurtosis provided better forecasts in some cases. The main influences on market volatility were the expected liquidity conditions, level of trade and spill-over effects from other financial markets. The spread was also found to have some explanatory power.

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¹ The views expressed are those of the authors and do not necessarily reflect those of the Bank of Jamaica.

1. INTRODUCTION

Given the small open and import dependent nature of the Jamaican economy, the exchange rate is probably the most important asset price. The exchange rate has been found to be an important element in the monetary transmission process in Jamaica² and movements in this price has a significant pass-through to consumer prices³. Against this background, understanding and forecasting exchange rate behaviour is important to monetary policy. More importantly, because of the thinness and volatility of the market, the policy makers focus on the information content of the short-term volatility. That is, while the medium to long-term outlook is important, policy makers are also concerned about exchange rate movements in the very short run in deciding intervention policy. Despite this, very few studies have attempted to model and forecast exchange rate dynamics in Jamaica, particularly in the very short run. This paper attempts to fill this gap.

Traditionally, exchange rates have been explained largely by macroeconomic variables – fundamentals. Market fundamentals as defined by Hodrick (1990) are economic variables that models with rational behaviour predict are the determinants of asset prices. In recent years, a number of models assessing the determinants of the nominal exchange rate have been advanced. In the flexible price monetary model for example⁴, the relative supplies of domestic and foreign currency determine the exchange rate. In this model prices adjust so that PPP holds and there is equilibrium in the goods market. Domestic and foreign currency assets are perfect substitutes and with perfect capital mobility UIP holds. Dornbusch (1976) introduced sticky prices and showed that unanticipated monetary disturbances lead to exchange rate overshooting. Relaxing the assumption of perfect asset substitutes and incorporating non-monetary assets, the portfolio balance approach shows how the equilibrium allocation of wealth is determined by exchange and interest rates. More recent models allow for the fact that domestic agents do hold foreign currency. In this setting, the greater the rate of substitution between both currencies the greater the deviation of expected future exchange rate from the spot rate.

Following the seminal paper of Meese and Rogoff (1983), however, the ability of economic fundamentals to predict exchange rates has been questioned, particularly in the short run⁵. In

² see Robinson and Robinson (1997), Allen and Robinson (2004)

³ see Robinson (2000a and 2000b) and McFarlane (2002)

⁴ see Frenkel (1976) and Musa (1976)

⁵ Meese and Rogoff (1983) in their empirical study of exchange rates concluded that for some assets, in particular exchange rate, prices and fundamentals are largely disconnected. Hallwood and MacDonald

particular, the random walk model was found to out perform the traditional structural models. This suggests that the exchange rate is influenced by factors other than macroeconomic fundamentals and as such these models do not adequately explain the short run characteristics of asset markets.

In seeking to address the shortcomings of the structural approach in explaining the dynamics of exchange rates there is a growing body of literature that focuses on the market microstructure with the use of high-frequency data.⁶ In this context, empirically, it accounts for the fact that many financial time series do not have a constant mean and constant variance, but exhibit volatility clustering, which is not explained by the standard theory. This approach encapsulates issues relating to information asymmetries, heterogeneity of participants and market configurations.

This paper adopts an eclectic approach in that it incorporates both market microstructure, as well as macroeconomic fundamentals in modelling and forecasting Jamaica's exchange rate at the daily frequency. The closest work is Walker (2002), who focused only on a limited set of microstructure variables in a linear GARCH model. We, however, account explicitly for the asymmetric response of the market to shocks and the significant leptokurtosis in the exchange rate. This, as it is known that the standard linear GARCH model is inadequate in the presence of these factors and as such this paper employs nonlinear asymmetric GARCH models. We also revisit the mixture of distribution of hypothesis theorem, which explores the relationship between volume and volatility⁷ and assess the significance of co-volatility across markets. Finally, we test the forecasting performance of the various models over a 30-day horizon.

The paper proceeds as follows. Empirical regularities of assets returns and the characteristics of the Jamaican foreign exchange market are discussed in section 2. Section 3 provides a brief review of the models of time varying heteroscedasticity followed by in-sample estimation results in section 4. The forecasting performance of the models is assessed in section 5, followed by some concluding comments in section 6.

(1994) suggested that exchange rate theory that provides a satisfactory and empirically consistent story of the exchange rate remains to be uncovered. However, Mark (1995) found fundamentals to have significant explanatory power over the long horizon.

⁶ With the use of high-frequency data it is hoped that the short-run volatility can be effectively modelled.

⁷ See Walker (2002) and Galati (2000).

2.0 EMPIRICAL REGULARITIES of the FOREIGN EXCHANGE MARKET

Asset markets are known to possess some common features or regularities. While these features are mostly found in stock markets, there is some evidence that they are present in foreign exchange markets. This section examines the presence of such regularities in the Jamaican foreign exchange market, which will determine the relevant model or statistical distribution for the exchange rate. We first discuss these common features and the international evidence and then examine whether they are present in the Jamaican data.

2.1 Empirical Regularities

2.1.1 Non-Normality

Many researches have found the empirical distribution of asset returns to be highly non-normal⁸. There is evidence of skewness (positive or negative), as well as excess kurtosis, meaning that asset returns are normally leptokurtic. The presence of excess kurtosis or thick tails in asset returns implies that estimations based on the assumption of identical and independently distributed (i.i.d.) errors are inappropriate for asset returns.

2.1.2 Volatility Clustering

The variance of speculative prices or asset returns is not constant over time. In fact asset prices are commonly characterised by what Mandelbrot (1963) described as volatility clustering – large changes followed by large changes and small changes followed by small changes⁹. Bollerslev, Engle and Nelson (1994) noted that volatility clustering is intimately related to the return series being characterized by thick tailed distributions. While some researchers regard this phenomenon as persistence of speculative activities, other attributes it to uncertainty or risk. Uncertainty about future market fundamental associated with current and expected fiscal and monetary policies, as well as corporate decisions, generate bouts of increased volatility. Connolly and Sivers (1999) concluded that volatility clustering is a natural result of a price formation process with heterogeneous beliefs across traders. As such, clustering is not attributed to an autocorrelated news-generating process around public information or news, such as macroeconomic developments or releases on firms' earnings.

⁸ See Fama (1965) and Kim and Kon (1994).

⁹ This is one feature of asset returns which structural models fail to capture.

Gokcan (2000) suggested that volatility is related to the stage of market development. Risk and returns characteristics of speculative prices in emerging markets are different from those in developed markets. That is, volatility in emerging markets are larger and more persistent than development markets. One explanation is the speed and reliability of information available to investors, which is associated with modes of telecommunication and possible accounting system in place. Developing economies often lag in terms of market development.

Volatility clustering or non-constant variance and thick tails are related. This as if the unconditional kurtosis of the innovations, e_t , is finite then the moment condition $E(e_t^4)/E(s_{e_t}^2)^2 \geq E(z_t^4)$ holds with strict equality only if s_t is constant. Further, there is strong evidence in the finance literature linking volatility in asset returns with higher order serial correlation¹⁰.

2.1.3 Non-trading Periods and Daily Seasonality

It is widely accepted that information accumulates during the period when financial markets are closed and is reflected in the price when the markets reopen. As such variances are higher following weekends and holidays. This leads to the observation of daily seasonality in asset returns also known as the days-of-the-week effect (DOW).

2.1.4 Regular Events

The release of important information is found to be associated with high volatility. Harvey and Huang (1992) for example find that foreign exchange volatility is higher when there is heavy central bank trading or there is a release of macroeconomic news. Patell and Wolfson (1979) document similar evidence for the stock market. The pattern of volatility during the trading day is also found to be predictable in that volatility is typically higher at the open and close of trading in both stock and foreign exchange markets¹¹.

2.1.5 Asymmetry and Leverage Effects

It is known that the magnitude of the response of asset prices to shocks depends on whether the shock is negative or positive. To illustrate this point Engle and Ng (1990) mapped the relationship between the conditional variance of asset returns to exogenous shocks which resulted in what they termed a news impact curve. They found evidence of asymmetry in stock returns. In

¹⁰ See Kim (1989)

¹¹ See Baillie and Bollerslev (1992).

an attempt to explain the asymmetry of volatility in speculative prices, Black (1976) posited that when stock price falls the value of the associated company's equity declines. As a consequence, leverage - the debt to equity ratio of the company- rises, thereby signalling that the company has become riskier. Increased risk is considered an indicator for higher volatility. Used in this context it is widely accepted that the statistically interpretation of Black's *leverage effect* implies that negative surprises increases predictable volatility in asset markets more than positive surprises. Kisinbay (2003) found asymmetry in stock returns but not in foreign exchange rate returns.

Another explanation of asymmetry is called the *volatility feedback hypothesis*¹². This was developed to explain stock price volatility. A negative shock to volatility increases the future risk premia. This would cause the stock price to fall if the future dividends are expected to remain the same. When applied to the foreign exchange market, a positive shock, which increases the volatility of the market, increases the risk of holding the currency. This induces a portfolio shift out of the currency, leading to a depreciation of the exchange rate.

2.1.6 Co-movement in Volatility

The earliest observation of correlation in volatility can be found in Black (1976) for stock markets. Harvey et al (1992) provide similar evidence for stock markets and Engel et al (1990) for the US bond market. The co-movement in volatility not only holds across different assets within a market but also across markets. For example, King et al (1994) and others have found co-movement across international markets.

2.1.7 Market Fundamental and Volatility

Evidence supporting a link between markets fundamental, such as interest rates, money supply and growth, and volatility in asset returns (in particular stock and exchange rates) has been very weak. Meese and Rogoff (1983) concluded that the prices of some assets, in particular exchange rate and economic fundamentals are largely disconnected. Frankel and Froot (1990) advance that unlike the exchange rate, changes in market fundamental are not as capricious. Notwithstanding this empirically results, there is still widespread support for the view that volatility in asset returns is closely tied to the health of the economy.

¹² See Campbell and Hentschel (1992)

2.1.8 Market Microstructure and Volatility

Following the failure of macroeconomic variables and fundamentals to explain volatility in speculative prices, microstructure variables have been identified as factors that can explain volatility. For the foreign exchange market, thinness indicated by volumes traded and the bid-ask spreads, market fragmentation measured by the degree of concentration, heterogeneous expectations and volatility spill over from other markets have all been considered¹³.

Empirical tests using microstructure variables find a positive relationship between volatility and spreads and volumes traded¹⁴. Spreads reflect the cost of transacting in foreign currency and as such will fluctuate in the same direction of risk of which volatility is the main indicator. Volume and volatility are said to be influenced by the same process of information arrival and as such should be positively correlated¹⁵. However, there is the view that relationship between volumes and volatility will depend on whether the market is fully developed as against an emerging market. In this context, Tauchen and Pitts (1983) argue that the relationship can be negative.

The more fragmented the market the higher the level of volatility. As a market becomes more concentrated, the greater the likelihood of the action of one investor influencing prices. This suggests a positive relationship between market concentration and volatility. This has implication for market efficiency. Speculative prices are also known to respond differently to expectation and contagion risks. With the increasing global integration of financial markets, the opportunity for greater diversification of risk arises. However, because of the varying levels of market integration, greater opportunities for arbitrage.

2.2 Characteristics of the J\$/US\$ Exchange Rate

This paper examines the daily dollar exchange rate of Jamaica¹⁶ over the period 2 January 1998 to 12 February 2003, a total of 1280 trading days after removing weekends and holidays. The plots of the autocorrelation function (ACF) and the partial autocorrelation function (PACF) shown in figure 1 suggest that the series is non-stationary, possibly with a drift component. The correlogram dies out slowly indicative of a long memory, or long-term dependence process. This suggests that there appears to be a persistent temporal dependency between observations over

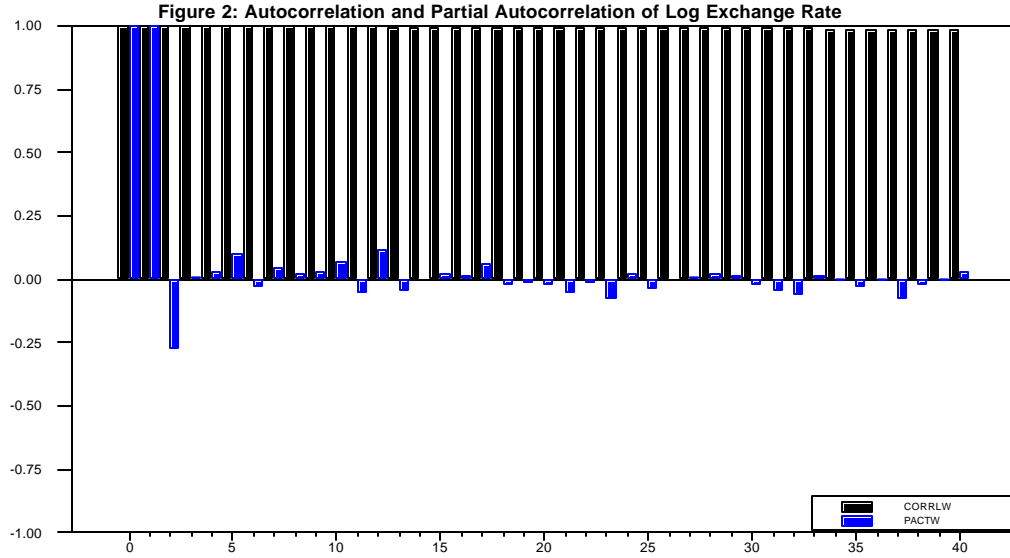
¹³ See Galati (2000) and Walker (2002) for an exposition on these relationships.

¹⁴ See Clark (1973) and Frankel and Froot (1990)

¹⁵ This relationship describes what Clark (1973) posits as the mixture of distribution hypothesis.

¹⁶ Dollar exchange rate is defined as units of notional currency per US dollar. The focus is on the average spot rate.

various displacements. The presence of a long memory process in the exchange rate series indicate that shocks to the exchange rate have some amount of permanence and the mean and variance of the series are time dependent¹⁷.



To more rigorously test for stationarity we use the Phillips and Perron (1988) test for unit root, which involves estimating the test regression.

$$s_t = \mathbf{m} + \mathbf{b}(t - T/2) + \mathbf{a}s_{t-1} + \mathbf{e}_t$$

where s_t is the log of the exchange rate and \mathbf{e}_t the innovation. The hypotheses are $H_o^1 : \mathbf{a} = 1$ and $H_o^2 : \mathbf{b} = 0, \mathbf{a} = 1$ which are tested using the $Z(t_a)$ and $Z(\Phi_3)$ ¹⁸. The computed test statistics are -1.24 and 1.09 for the $Z(t_a)$ and $Z(\Phi_3)$ respectively, against the 5% critical values of -3.41 and 6.25 respectively. Thus the unit root hypothesis cannot be rejected.

It is known, however, that these tests do not account for the presence of near unit roots or a long memory process. That is, these tests cannot detect an order of integration, d , which is less than unity. In which case the series is said to be fractionally integrated and has to be differenced by

¹⁷ The presence of long memory process in the asset prices contradicts the weak form of market efficiency, which stipulates that, conditioned on historical information, future returns or movements in prices are unpredictable.

¹⁸ See Perron (1988) for the precise form of these statistics.

$d < 1$ times to be made stationary. As such we also employ the Geweke and Porter-Hudak (1983) spectral regression test for fractional integration.

A time series \mathbf{y} follows an autoregressive fractionally integrated moving average process of order (p, d, q) (ARFIMA (p, d, q)) if

$$\Phi(L)(1-L)^d(y_t - \mathbf{d}) = \Theta(L)\mathbf{e}_t, \quad \mathbf{e}_t \sim iid(0, \mathbf{S}_3^2) \quad (1)$$

where L is the lag operator, $\Phi(L) = 1 - \mathbf{f}_1 L - \dots - \mathbf{f}_p L^p$, $\Theta(L) = 1 + \mathbf{J}_1 L + \dots + \mathbf{J}_q L^q$, and $(1-L)^d$ is the fractional differencing operator defined by

$$(1-L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \quad (2)$$

with $\Gamma(\cdot)$ denoting the gamma, or generalized factorial, function. The stochastic process \mathbf{y} is both stationary and invertible if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < 0.5$. When $d \in (-0.5, 0.5)$ and $d \neq 0$ that the autocorrelation function of an ARFIMA process decays hyperbolically to zero as $j \rightarrow \infty$, at a much slower rate than the exponential decay of a stationary ARMA process (i.e., $d=0$).

Geweke and Porter-Hudak (1983) suggested a semi-parametric procedure to obtain an estimate of the fractional differencing parameter d based on the slope of the spectral density function around the angular frequency $\mathbf{x} = 0$. More specifically, let $I(\mathbf{x})$ be the periodogram of \mathbf{y} at frequency \mathbf{x} defined by

$$I(\mathbf{x}) = \frac{1}{2pT} \left| \sum_{t=1}^T e^{i\mathbf{x}t} (y_t - \bar{y}) \right|^2. \quad (3)$$

Then the spectral regression is defined by

$$1n\{I(\mathbf{x}_I)\} = \mathbf{b}_0 + \mathbf{b}_1 1n\left\{ \sin^2\left(\frac{\mathbf{x}_I}{2}\right) \right\} + \mathbf{h}_I, \quad I = 1, \dots, \nu \quad (4)$$

where $\mathbf{x}_I = \frac{2pI}{T}$ ($I = 0, \dots, T-1$) denotes the harmonic ordinates of the sample, T is the number of observations, and $\nu = g(T) \ll T$ is the number of harmonic ordinates included in the spectral regression. The negative of the OLS estimate of the slope coefficient in (4) provides and

estimate of d . The plot of the estimates of d is shown in Figure 2. The estimates appear to settle down around 0.83, thereby supporting the hypothesis of fractional integration¹⁹.

The immediate implication of these results is that the exchange rate follows a random walk and hence the use of a martingale or random walk model for short-run exchange rate movements. A random walk model of the exchange rate changes, however, may not be appropriate in the presence of conditional heteroscedasticity. In this context we study the properties of the estimated innovations from a simple random walk model with drift. We use both the first difference, which corresponds to a single unit root and the fractional differenced series. With respect to the later, the series was differenced 0.83 times using the binomial operator in equation 2. The plots of both differenced series are shown in Figure 3 and the relevant statistics in Table 1.

Table 1: Summary Statistic for the Return Series

Series	Mean (%)	Variance (%)	Skewness	Excess Kurtosis	Jarque-Bera Normality Test ¹	Lung-Box Test Q(40)	Lung-Box Test Q ² (40)
1st difference	0.000267	0.000004	0.696	8.38	1332.031 (P=0.0)	235.36	478.22
fractional diff.	0.0047	0.000014	1.773	4.11	1091.1(P=0.0)	22153.61	17121.86

The graph of the return series exhibits bouts of intense volatility followed by periods of tranquillity, which is consistent with the volatility-clustering hypothesis in the finance literature. Periods of intense volatility are followed by further observations of high volatility. The volatility in the Jamaican foreign exchange market reflects the stage of market development as Gokcan (2000) suggests, in combination with a latent perception of risk or low confidence.

Although the number of players, the depth of the market and consequently the degree of competition has increased significantly over the sample period, the market is largely dominated by relatively few large players. Further market trades are segmented between large contract transactions and smaller over the counter trades. The trading infrastructure is also relatively under-developed. This framework doesn't permit the efficient dissemination of information

¹⁹ In the absence of a long memory process the plot of the coefficients from GPH process should trending towards 1. When this does not happen, fractional differencing of a series will result in a stationary process.

Figure 2: Degree of Integration

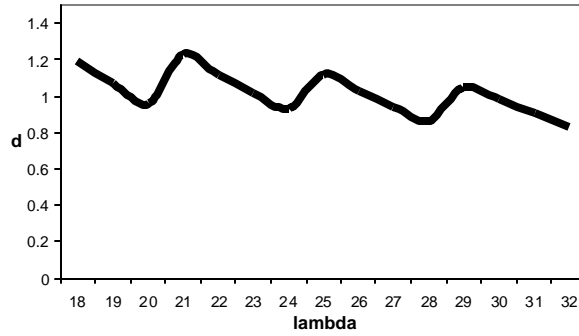


Figure 3A

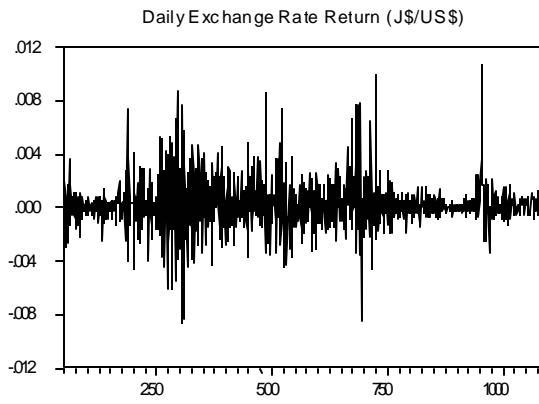


Figure 3b

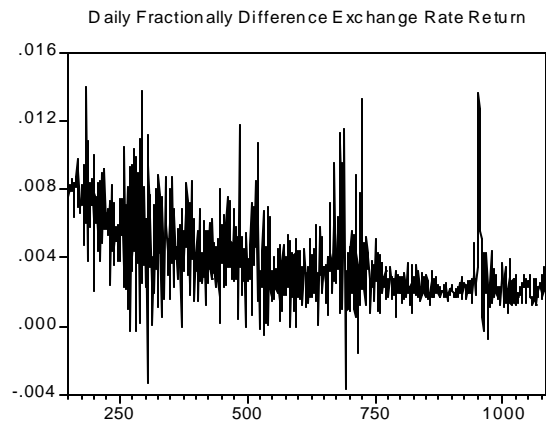


Figure 4a

Distribution of fractionally Difference Return Series

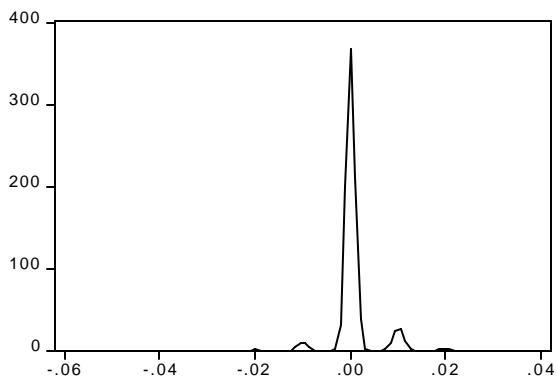
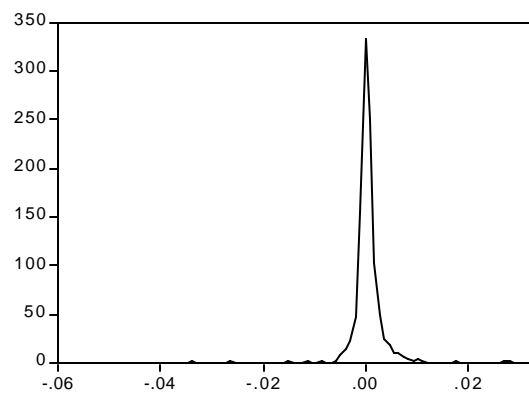


Figure 4b

Distribution of Difference Return Series



among traders and end users. Thus there may well be an autocorrelated news-generating process from time to time in the market²⁰.

Within a thin market any negative news or fad, however generated, can generate large swings in a context where the perceived risk of holding Jamaica Dollars may (or may not) be higher than what is warranted by fundamentals. The level of confidence reflects historical shocks, particularly in the early part of the 1990s, which from the unit root analysis, tend to be long lived.

The reported statistics in Table 2 indicate the presence of significant excess kurtosis, particularly for the first differenced series. In fact, the excess kurtosis is larger than that found for the currencies of countries with developed markets in Baillie and Bollerslev (1989). The Ljung and Box (1978) test statistic for the k th-order serial correlation suggest significant higher order autocorrelation. The distribution is significantly skewed towards the left, particularly for the fractionally differenced series. Skewness indicates non-normality, while the relatively large kurtosis suggests that distribution of the return series is leptokurtic, signalling the necessity of a peaked distribution to describe this series. The Ljung and Box $Q^2(k)$ statistics rejects the hypothesis of conditional homoscedasticity. The kernel plots (Figures 4a and 4b) of the unconditional distribution confirms that the unconditional distribution of the return series not Gaussian.

In summary, the preceding analysis indicates that the empirical distribution of returns in the foreign exchange rate market is non-normal, with very thick tails. The leptokurtosis reflects the fact that the market is characterised by very frequent medium or large changes. These changes occur with greater frequency than what is predicted by the normal distribution. The empirical distribution confirms the presence of a non-constant variance or volatility clustering. The degree of leptokurtosis when compared to other markets may be reflective of the thinness of the Jamaican foreign exchange market, where small movements or shocks tend to get magnified overtime. It may also reflect uncertainty regarding Government policies and other economic fundamentals

The presence of long memory implies that shocks are long lived. The more significant result however is that of the asymmetry of the distribution, which implies that positive shocks (i.e. shocks that lead to a depreciation) are more likely than negative shocks. The response of the

²⁰ See Walker (2002) for further detailsa.

market may however differ depending on the nature of the shock (i.e. positive or negative). The thick left tail means that the density of a return's distribution is asymmetric with a sharper fall on the right tail, which is an indication of the fact that market declines occur with a greater frequency than increases. In this context, the risk of holding long position in foreign exchange is relatively small. In other words, short positions are relatively more expensive.

3. MODELS OF TIME VARY CONDITIONAL HETEROSCEDASTICITY

This section present an overview of the models used given with the statistical properties of exchange rate returns found above, namely asymmetry and significant fat tails. Generally, time varying heteroscedasticity is modelled by the linear GARCH model of Bollerslev (1986) i.e.

$$\Delta s_t = b_0 + \mathbf{e}_t, \quad \mathbf{e}_t | \Omega_{t-1} \sim D(0, h_t)$$

where,

$$h_t = w + \sum_{i=1}^p \mathbf{f}_i \mathbf{e}_{t-i}^2 + \sum_{j=1}^q \mathbf{b}_j h_{t-j} \quad (5)$$

where w , \mathbf{f}_j and \mathbf{b}_j are constant and non-negative parameters. This specification allows for the conditional variance to be dependent on past information, which will induce variability over time. More specifically, the conditional variance is explained by past shocks and past variances²¹. The key features of this specification are that if $p=0$, the process reduces to an ARCH (q) process and \mathbf{e}_t a white noise process when $p = q = 0$. To induce stationarity and to prevent negative variances $\sum_{i=1}^p \mathbf{f}_i + \sum_{j=1}^q \mathbf{b}_j < 1$ ²² must hold.

Generally, the linear GARCH (p, q) model, based on the conditional normal distribution, captures thick tails and other stylised facts such as non-trading periods and regular events. Notwithstanding the apparent success of linear GARCH models, Engle and Ng (1991), Bollerslev, Chou and Kroner (1992) and other leading researchers have suggested that there are features of the data that these models cannot capture. For example, it doesn't always account for

²¹ Engle and Ng (1991) examined the implied relationship between past errors and the conditional variance. The graphical representation of this relationship is relationship is termed the news impact curve. The exact shape of this curve is dependent on the specification of h_t .

²² See Bollerslev (1986) for a comprehensive discussion on the need for these restrictions.

significant fat tailedness in the unconditional distribution²³. Further, it is found to be deficient in correcting bias in the forecast and forecast error variance associated with a skewed distribution. Against this background, there have been a number of extensions the GARCH (p, q) model to explicitly account for the skewness. We have restricted our analysis to the more popular models of asymmetric volatility. These includes, the exponential GARCH (EGARCH) model, Glosten, Jaganathan, and Runkle (1992) GJR-GARCH model, asymmetric power ARCH (APARCH), Zakoian (1994) threshold ARCH (TARCH). The TS-GARCH advanced by Taylor (1986) and Schwert (1990), the t-GARCH and the generalized version of Higgins and Bera (1992) non-linear ARCH (NGARCH) are included to capture the information content within the thick tails of the return distribution.

3.1 Models of Fat Tails

3.1.1 t-GARCH

The most common approach used when the error distribution tends to have significantly fatter tails than the normal distribution is to adopt the Student-*t* distribution. This gives to the t-GARCH model where the degrees of freedom are also estimated. The functional form of h_t remains the same, however the normal density function in the log likelihood is replaced by the Student-*t*.

3.1.2 NGARCH

The NGARCH model is a generalization of the Higgins and Bera (1992) non-linear ARCH, which only contained ARCH lags. The NGARCH has the following structure:

$$h_t = w + \sum_{i=1}^p \mathbf{f}(\mathbf{e}_{t-i} + \mathbf{g}h_{t-1}^{1/2})^2 + \sum_{j=1}^q \mathbf{b}_j h_{t-j} \quad (6)$$

In this model the asymmetric effect depend upon the standard deviation.

3.1.3 TS-GARCH

The TS-GARCH model developed by Taylor (1986) and Schwert (1990) is another popular model used to capture the information content in the thick tails, which is common in the return distribution of speculative prices. The specification of this model is based on standard deviations and is as follows:

$$(h_t^{1/2}) = w + \sum_{i=1}^p (\mathbf{f}_j | \mathbf{e}_{t-i} |) + \sum_{j=1}^q \mathbf{b}_j (h_{t-j}^{1/2}) \quad (7)$$

²³ See for example Baillie and Bollerslev (1989) and Hseih (1988).

3.2 Non-linear GARCH models

3.2.1 EGARCH

The exponential GARCH (EGARCH) model advanced by Nelson (1991) is the earliest extension of the GARCH model that incorporates asymmetric effects in returns from speculative prices. In short, the h_t is an asymmetric function of past \mathbf{e}_t 's and is defined as follows:

$$\log(h_t) = w + \sum_{i=1}^p \mathbf{f}_j \left[\frac{|\mathbf{e}_{t-i}|}{\sqrt{h_{t-i}}} - \sqrt{2/p} \right] + \mathbf{g} \frac{\mathbf{e}_{t-1}}{\sqrt{h_{t-1}}} + \sum_{j=1}^q \mathbf{b}_j \log h_{t-j} \quad (8)$$

where w , \mathbf{f}_j , \mathbf{g} and \mathbf{b}_j are constant parameters. Unlike the GARCH (p, q), the form of the EGARCH (p, q) equation indicates that the conditional variance is an exponential function, thereby removing the need for restrictions on the parameters to ensure positive conditional variance. The asymmetric effect of past shocks is captured by the \mathbf{g} coefficient, which is usually negative, that is, ceteris paribus positive shocks generate less volatility than negative shocks. This feature permits the capture the sign effect by allowing positive and negative innovations to have different effects on the volatility. If $\mathbf{g} = 0$, positive and negative shocks have the same effect on volatility. The size effect is captured by \mathbf{f}_j and is expected to be positive. Shocks are measured relative to its standard deviations. The use of absolute shocks and logs in this parameterisation allows us to capture the size effect, in that it increases the impact of large shocks on the next period conditional variance.

3.2.2 GJR-GARCH

the GJR-GARCH (p, q) model is another volatility model that allows asymmetric effects. This was introduced by Glosten, Jagannathan and Runkle (1993). The general specification of this model is of the form:

$$h_t = w + \sum_{i=1}^p (\mathbf{f}_j \mathbf{e}_{t-i}^2 + \mathbf{g} S_{t-i}^- \mathbf{e}_{t-i}^2) + \sum_{j=1}^q \mathbf{b}_j h_{t-j} \quad (9)$$

where the only difference from the general GARCH (p, q) model is $\sum_{i=1}^p (\mathbf{g} S_{t-i}^- \mathbf{e}_{t-i}^2)$. S_t^- is a dummy variable that is equal to 1 if $\mathbf{e}_{t-i} < 0$ and zero otherwise. It is this extra term, which allows for the asymmetric effect, as the impact of \mathbf{e}_{t-i}^2 on h_t depend on whether the shock is negative or positive. In the event that a negative shock is realized then the impact on volatility will be

$\mathbf{f}_j + \mathbf{g}$ and \mathbf{f}_j in when the shock is positive. The parameter \mathbf{g} , which captures the asymmetric effect, is expected to be positive.

3.2.3 APARCH

The asymmetry power ARCH (APARCH) model of Ding, Granger and Engle (1993) allows for asymmetric effects of shocks on the conditional volatility in a more general framework. Formally, the general specification of the APARCH (p, q) model is:

$$(h_t^{1/2})^d = w + \sum_{i=1}^p (\mathbf{f}_j |\mathbf{e}_{t-i}| - \mathbf{g}_i \mathbf{e}_{t-i})^d + \sum_{j=1}^q \mathbf{b}_j (h_{t-j}^{1/2})^d \quad (10)$$

where w , \mathbf{f}_j , \mathbf{d} and \mathbf{b}_j are constant, positive parameters, and $-1 < \mathbf{g}_i < 1$.

Asymmetry or leverage effect in this model is captured by the \mathbf{g}_i term. For an APARCH (1,1) model, when $\gamma > 0$, negative shocks lead to higher volatility and vice versa. APARCH (p, q) models of asymmetric differs from other GARCH type volatility models with the introduction of the power term, \mathbf{d} , which is to be estimated. The introduction and estimation of the power term is an attempt to account for the true distribution underlying volatility. The idea behind the introduction of a power term arose from the fact that in modelling financial data, the assumption of normality, which restricts \mathbf{d} to either 1 or 2, is often unrealistic due to significant skewness and kurtosis²⁴. Allowing \mathbf{d} to take the form of a free parameter to be estimated removes this arbitrary restriction.

3.2.4 TARCH

The threshold ARCH or TARCH (p, q) model introduced by Zokoian (1994) is very similar to the GJR-GARCH model with the exception that it is the conditional standard deviation that is modelled and not the conditional variance. In this regard the parameterization is similar to equation (5) with the difference being that h_t is replaced with the square root of h_t .

²⁴ Under the assumption of normality, the entire distribution can be defined by the first two moments. That is, \mathbf{d} is either 1 or 2. When $\mathbf{d} = 1$, the APARCH model evaluates the standard deviation. $\mathbf{d} = 2$, the variance is modelled.

3.3 Co-Volatility

Multivariate GARCH models, first introduced by Kraft and Engle (1983), allows for co-movements between assets. Subsequent studies have advanced various modifications to the original specifications. The more popular models include the VECH model by Bollerslev, Engle and Wooldridge (1988), the CCC model of Bollerslev (1990), Engle, Ng and Rothschild (1990) factor model and the BEKK model of Engle and Kroner (1995). The basic intuition behind all these models is that conditional covariance matrix H_t of the \mathbf{e}_t 's is dependent on past information (Ω_{t-1}). Where the models differ is in relation to the parameterisation of H_t .

This paper uses the BEKK multivariate framework, which is given as²⁵:

$$\mathbf{m}_t = E(y_t | \Omega_{t-1}) = \Gamma X_t + \mathbf{e}_t \quad \mathbf{e}_t | \Omega_{t-1} \sim D(0, H_t)$$

and,

$$H_t = C + A \mathbf{e}_{t-1} \mathbf{e}_{t-1}' A' + B H_{t-1} B' \quad (11)$$

where C, A and B are all N*N parameter matrices. y_t is a vector of returns on N assets during period t, and \mathbf{m}_t , the conditional mean vector. The BEKK allows the conditional covariance matrix to be determined by the outer product matrices of the vector past return shocks. Because the number of parameters in the BEKK model is large, $(5/2N^2 + N/2)$, it is necessary it place restrictions on the off diagonals to ensure interpretability.

3.4. Model Selection

Selecting the most appropriate model, particularly when the true underlying distribution is unknown can be complicated. Most researchers tend to rely on the traditional Akaike and Schwartz information criteria. However their statistical properties and hence reliability is unknown in the context of time varying volatility. As such, model selection criteria typically focus on the estimation of loss functions for alternative models. When applied to models with time varying volatility such loss functions depend on the squared residuals and the variance. This paper uses three such measures:

$$L_2 = \sum_{t=1}^T (\mathbf{e}_t^2 - \mathbf{s}_t^2) \mathbf{s}_t^{-4}$$

²⁵ See Gau (2001) for a simple exposition on the more popular multivariate GARCH models.

$$L_3 = \sum_{t=1}^T (\ln(\mathbf{s}_t^2) + \mathbf{e}_t^2 \mathbf{s}_t^{-2})$$

and

$$L_4 = \sum_{t=1}^T (\ln(\mathbf{e}_t^2 \mathbf{s}_t^{-2}))^2$$

4. IN – SAMPLE RESULTS

4.1 Data

Changes in the monetary base are used as an indicator of changes in Jamaican dollar liquidity conditions. Due to data availability this was the main macroeconomic fundamental variable used. A more appropriate measure of changes in liquidity conditions would be the movement in the overnight or inter-bank rates. However, data was not available for the full sample period. The microstructure variables employed include, trading volumes, bid-ask spread and dummies representing each day of the week. The inclusion of volumes captures the thinness of the market. As stated before the period of study is from 2 January 1998 to 12 February 2003.

Tables 2 and 3 document the results for the fractional and 1st differenced series respectively²⁶, while the model selection criteria are given in Tables 4 and 5. With the exception of the APARCH model in Table 2, the gammas are significant, confirming the importance of asymmetry in the exchange rate volatility. The results are weaker for the series, which used the assumption of a single unit root. Only the coefficients in the GJR and the APARCH models have the correct sign, which implies that negative news lead to greater volatility in the Jamaican foreign exchange market.

The information criteria suggest that the model to use is the GJR. This is further supported by the model selection criteria based on the loss functions. There is also support for the TS-GARCH model, which treats with fat tails. Generally, for both the fractional and unit root series, the asymmetric GARCH models perform best in terms of minimizing the loss functions. This contrasts with those of Kisinbay 2003 who found asymmetry in stock returns but not in foreign exchange returns. The difference in results may lie in the microstructure of the markets studied.

²⁶ The NGARCH models did not converge and as such were excluded from this section. We report the results from simple GARCH models in the appendix.

Kisinbay look at the deutschmark/US dollar and the yen US dollar markets which are considered to very developed compared with the Jamaica / US dollar market.

Further, we postulate that this asymmetry reflects the volatility feedback hypothesis. That is any shock to exchange rate volatility, which itself is an indicator of risk, increases the currency risk premia (as well as the cost of international trade), which unless compensated for by a higher interest rate differential, leads to greater volatility as investors freely adjust the portfolios to hedge against currency risk.

Mean return for each day of the week is largely positive and statistically significant. The reported chi-squared statistic which test for equality of return across the days of the week, suggest that daily mean returns are significant different from each other. Though the deviations from Monday's returns are on average 0.1 per cent, the evidence confirms the presence of a DOW effect in Jamaica's foreign exchange returns. Worthy of note is the negative and significant coefficient on Thursday under the APARCH model shown in Table 5 suggesting that returns are the highest on Thursdays. The presence of DOW effect in asset returns is evidence against market efficiency, as investors are able to predict with reasonable certainty movements in the exchange rate. In fact, by the virtue of the deviations from Monday's return being positive, suggest that returns are generally largest on Mondays. A large Monday return is evidence that information gained on non-trading days are important in Jamaica's foreign exchange rate. To the policy markers, this is an indication that care must be taken when interpreting depreciation in the exchange rate on Mondays.

Mean returns in Table 2 decline as the volumes traded increase, however mixed results are obtained for models reported in Table 3. There is also evidence that the market respond to expected changes in the monetary base. Return is generally higher when the market expects expansionary change in the base. This condition could be a result of speculative tendencies associated to with excess Jamaica dollar liquidity.

Table 2: Parameter Estimates for the GARCH Models (Fractional)

Variables	EGARCH	TGARCH	APARCH	GJR- GARCH	TS- GARCH	TARCH
Mean						
Constant	0.03 (21.43)	0.07 (48.14)	0.05 (3.38)	0.05 (32.76)	0.12 -6.51	0.04 (29.37)
Tuesday	0.00 (7.96)	0.00 (18.32)	0.00 (20.20)	0.00 (3.95)	0.00 90.69	
Wednesday	0.00 (5.09)	0.00 (21.93)	0.00 (39.50)	0.00 (4.76)	0.00 (1.42)	
Thursday	0.00 (3.83)	0.00 (11.55)	0.00 (39.03)	0.00 (3.26)	0.00 (-0.74)	
Friday	0.00 (3.38)	0.00 (7.09)	0.00 (32.22)	0.00 (1.64)	0.00 (-2.53)	
Dlbase{+1}	0.00 (0.85)	-0.03 (-13.07)	0.01 (6.40)	0.01 (2.41)	0.01 (0.33)	0.00 (0.72)
Lvolumes	0.00 (-19.61)	0.00 (-47.31)	0.00 (-0.24)	0.00 (-31.58)	-6.50 (-0.01)	0.00 (-27.40)
Variance						
Constant	-0.08 (-0.13)	0.00 (5.26)	0.02 (0.58)	0.00 (11.69)	0.00 (1.71)	0.00 (12.84)
ARCH(Alpha 1)	0.92 (57.47)	0.34 (20.23)	0.03 (0.46)	-0.14 (-2.05)	0.00 (4.86)	0.00 (11.6)
GARCH (beta 1)	0.29 (6.12)	0.24 (13.74)	0.88 (64.95)	0.05 (2.51)	0.02 (4.43)	0.00 (13.64)
Gamma1	0.14 (5.73)		0.99 (1.20)	0.10 (3.04)		-0.85 (-22.80)
Delta			0.07 (0.42)			
Spread		0.00 (14.25)	0.07 (0.06)	0.00 (7.41)	-0.01 (-1.34)	0.00 (1.80)
Lvolumes	-0.29 (-6.45)	0.00 (-14.37)		0.00 (-12.83)	0.00 (-12.46)	0.00 (-12.86)
Lvolumes {+1}		0.00 (2.42)				
Lwsxr {+1}	0.98 (4.27)			0.00 (-8.22)	0.00 (5.90)	
Dlbase						0.00 (0.66)
SBC	-330.37	-330.37	2705.40	-326.90	-330.40	2753.04
AIC	-349.42	-349.40	2769.60	-347.42	-349.43	2703.62
Chi-Squared (4)	8056.2 [0.0]	427.8 [p=0.0]	78.26.7 [0.0]	123.9 [0.0]	239.9 [0.0]	
No. Observations	1034.00	1034.00	1034.00	1034.00	1034.00	1034.00
Function Value	6023.77	4946.13	-22649.10	5641.61	3711.84	5766.78

Table 3: Parameter Estimates for GARCH Models (1st Difference)

Variables	EGARCH	TGARCH	APARCH	GJR-GARCH	TS-GARCH	TARCH
Mean						
Constant	0.00 (-4.45)	0.00 (3.15)	0.05 (26.52)	0.00 (-3.63)	0.00 (0.96)	0.00 (0.23)
Tuesday	0.00 (7.09)	0.00 (6.934)	0.00 (12.83)	0.00 (6.62)	0.00 (6.50)	0.00 (0.42)
Wednesday	0.00 (4.38)	0.00 (4.71)	0.00 (7.38)	0.00 (4.23)	0.00 (7.76)	0.00 (0.11)
Thursday	0.00 (3.33)	0.00 (3.63)	-0.03 (-25.23)	0.00 (2.90)	0.00 (4.30)	0.00 (0.01)
Friday	0.00 (4.08)	0.00 (4.65)	0.00 (11.89)	0.00 (3.84)	0.00 (5.72)	0.00 (0.52)
Dlbase{+1}		0.01 (2.24)	0.00 (4.49)	0.00 (1.73)	0.00 (0.74)	0.00 (4.61)
Dlbase	0.00 (1.16)					
Lvolumes	0.00 (3.07)	0.00 (-3.59)	0.00 (-21.77)	0.00 (2.53)	0.00 (-0.85)	0.00 (-0.22)
Variance						
Constant	-3.45 (-3.66)	0.00 (0.88)	-0.03 (-1.88)	0.00 (1.13)	0.00 (0.53)	0.01 (4.59)
ARCH(Alpha 1)	0.83 (42.27)	0.28 (8.22)	0.02 (1.39)	0.15 (2.13)	0.01 (7.71)	0.00 (-0.20)
GARCH (beta 1)	0.53 (13.60)	0.48 (12.82)	0.87 (68.53)	0.76 (40.49)	0.03 (7.60)	0.00 (0.72)
Gamma1	-0.01 (-0.32)		-0.57 (-4.43)	0.06 (1.59)	0.01 (0.003)	
Delta			0.20 (3.63)			
Spread	67.35 (5.60)	0.00 (5.82)	0.84 (2.19)		0.03 (2.84)	0.00 (0.22)
Lvolumes	-0.48 (-7.42)	0.00 (-8.12)	0.00 (-2.12)	0.00 (-3.35)	0.00 (5.93)	0.00 (-1.25)
Lvolumes {+1}		0.00 (-4.10)				
Lwsxr {+1}	2.34 (6.48)	0.00 (10.78)	0.02 (2.23)	0.00 (2.38)	0.00 (-4.39)	0.00 (-6.39)
SBC	-331.32	-330.18	-326.72	-330.37	-333.65	-330.18
AIC	-351.80	-350.70	-348.70	-350.70	-352.70	-350.70
Chi-Squared (4)	2679.7 [0.0]	145.3 [p=0.0]	0.75 [0.9]	0.00001 [1.0]	87.81 [0.0]	0.22 [0.9]
No. Observations	1034.00	1034.00	1034.00	1034.00	1034.00	1034.00
Function Value	6368.75	5946.95	6338.47	6333.29	3825.76	509.00

Notes: t-Statistics are reported in parentheses. Chi-squared values correspond to the joint F-statistic, which test whether the coefficients on the days of week are significantly different from zero. The maximum likelihood function value is also reported.

Table 4: Model Selection: Fractional Models

Loss Function	TGARCH	EGARCH	GJR	TS	TARCH	APARCH
L ₂	2669.67	1064.33	880.09	999.71	1028.80	1989.47
L ₃	-9724.66	-12304.00	-12796.10	-7434.43	-6860.16	-10728.45
L ₄	39288.29	18206.21	12682.29	34452.77	66762.03	N/A

Table 5: Model Selection: 1st Difference Models

Loss Function	TGARCH	EGARCH	GJR	TS	TARCH	APARCH
L ₂	3245.81	1023.45	768.48	833.00	1033.79	1445.93
L ₃	-13037.60	-11977.50	-13363.10	-11946.00	-6777.06	-11554.00
L ₄	4816.16	30910.60	15229.97	19348.00	N/A	N/A

A positive relationship is noted between spreads and volatility. One interpretation of this relationship is the risk of holding US dollar particularly in period of uncertainty, is not easily diversified, and, as a means of offsetting possible losses, the spread will rise. This could be reflective of the extent to which the foreign exchange market is largely concentrated in one currency. Therefore, higher spread in volatile periods reflects compensation for additional risk faced by holders of foreign currency. The largely negative contemporaneous relationship between volumes and volatility suggest one of three things; 1) either the local foreign exchange market is still in a developmental stage and as such any increase in participants may affect price; 2) it is volumes traded relative to demand that that dictates price variability – the more liquid the market, the less the likelihood of extreme volatility; 3) the market is often characterised by prolonged bouts of severe volatility, which normally results in depreciation. Should the third scenario obtain holders of foreign currency would benefit from lowering volumes traded in the volatile period. High volatility may also discourage existing investors and new entrants to the market and as such may result in lower volume²⁷. Expectation regarding future volumes are important for volatility, however, both set of results provide different signs.

The results in Tables 2 and 3 suggest that participants in the foreign exchange market build into today's trading expectations regarding the actual level of future exchange rate. The positive

²⁷ See Pagano (1989).

coefficient reported in both tables for the EGARCH, TGARCH, APARCH and the GJR in Table 5 suggests that as the market approaches this expected future rate, volatility will tend to increase. One explanation could be that trades are executed at the reservation price and or investors have some notion of a threshold rate, therefore, attempts by speculators to push the market beyond this rate will be met with strong resistance. However, with regard to the negative coefficient reported on this variable under the GJR model in Table 4 and the TS-GARCH and TARARCH of Table 5, one possible interpretation is that the exchange is deemed to be overvalued. When this happens a move to the equilibrium rate would result in lower levels of volatility.

4.2 Multivariate GARCH

Tables 6 and 7 show the results from estimating a trivariate GARCH model using the daily local exchange rate, USD/Pound exchange rate and the private money market interest rate over the sample period 7 February 2002 to 12 February 2003, a total of 250 trading days. The parameters from the multivariate GARCH model, which includes the fractionally difference exchange rate series, shown in Table 6, are mostly statistically significant. The mean equation suggests that, excluding the interest rates which represent the money market, a simple random walk model captures the dynamics of the variables under consideration. While all three markets are affected by shocks and volatility originating within the respective markets, there is also strong evidence supporting the existence of contagion. The higher the level of past foreign exchange rate volatility and shocks, the higher the level of current volatility. The significance of past shocks validates the long memory and volatility-clustering feature of foreign exchange return. In addition, the existence of this feature suggests that endemic to this market is a high degree of predictability. One possible explanation for this phenomenon is that money holdings (local or foreign) and money market instruments are competing assets. This also speaks to inefficiencies, in particular, the rate of information arrival and the extent to which the market is concentrated. In a very concentrated market it may not be possible to diversify to minimise exposure to past shocks.

Shocks originating in the money and cross rate markets increase volatility in the foreign exchange market. Past volatility in the money market has no effect on volatility in the foreign exchange market, however, volatility in the cross rate market tends to provide a stabilizing effect on both the money and foreign exchange markets.

Table 6:Parameter Estimates for Multivariate
GARCH (Fractional)

Parameters	Estimates	Standard Errors	t-Statistic
Mean			
constant 1	0.00	0.00	31.51
constant 2	-0.03	0.01	-2.94
constant 3	-0.09	0.19	-0.48
Variance			
constant 1	0.47	0.09	5.50
constant 2	0.01	0.03	0.27
constant 3	0.35	0.04	9.33
constant (1,2)	0.00	0.00	-0.11
constant (1,3)	0.00	0.00	0.42
constant (3,2)	0.01	0.00	7.16
error 1	0.78	0.09	8.38
error 2	1.07	0.09	11.78
error 3	-0.42	0.14	-2.96
error (1,2)	0.00	0.00	3.55
error (1,3)	0.00	0.00	2.65
error (3,2)	-0.01	0.00	-2.94
variance 1	0.00	0.00	3.64
variance 2	0.13	0.01	16.28
variance 3	2.64	0.19	13.87
variance (1,2)	0.00	0.00	-4.69
variance (1,3)	0.00	0.00	0.79
variance (3,2)	-0.03	0.02	-2.11
No. of Observation	222.00		
Function value	1373.9		
variable 1 = dlwsxr			
variable 2 = dlcross rate			
variable 3 = dir			

Table 7: Parameter Estimates for Multivariate GARCH (1st Difference)

Parameters	Estimates	Standard Errors	t-Statistic
Mean			
constant 1	0.00	0.00	0.82
constant 2	-0.01	0.01	-0.60
constant 3	0.08	0.23	0.34
Variance			
constant 1	0.00	0.00	0.09
constant 2	0.19	0.00	80.29
constant 3	4.74	0.21	22.78
constant (1,2)	0.00	0.00	-0.38
constant (1,3)	0.00	0.00	-0.44
constant (3,2)	-0.13	0.01	-9.97
error 1	0.31	0.06	4.85
error 2	-0.06	0.06	-1.08
error 3	0.04	0.04	0.83
error (1,2)	0.00	0.00	0.02
error (1,3)	0.00	0.00	1.33
error (3,2)	0.01	0.00	5.61
variance 1	1.00	0.41	2.47
variance 2	0.10	0.02	5.73
variance 3	-0.04	0.15	-0.26
variance (1,2)	0.00	0.00	0.71
variance (1,3)	0.00	0.00	0.06
variance (3,2)	-0.01	0.01	-1.28
No. of Observation		222.00	
Function value		1256.7	
variable 1 = dlwsxr			
variable 2 = dlcross rate			
variable 3 = dir			

For the model that uses the log change of the exchange, the parameters from the mean and variance equations are largely statistically insignificant. Worthy of note is the fact that volatility in Jamaica's foreign exchange market is influenced by past shocks as well as past volatility. For the USD/Pound market, current volatility is found to be affected by past volatility but not past shocks.

5. FORECASTS

In evaluating the forecasting power of the various GARCH models, the measure of volatility is important. Following Chong et al (1999) the following measure of the 'true' unconditional volatility for the foreign exchange market is used

$$s_t^2 = (r_t - \bar{r})^2$$

where s_t^2 is the unconditional volatility, r_t , is the actual daily return for day t , and \bar{r} is expected return for day t . The expected return over ten days is measured by calculating the arithmetic average of daily returns from day one to day nine. The expected return on day eleven is calculated by taking the arithmetic average daily returns from day two to day ten. This is repeated for the entire forecast period. Squaring the difference between the actual and moving average returns generates the implied volatility indicated by the above equation.

In order to obtain the one-period ahead forecast error for the different GARCH models, the following equation is employed

$$m_{t+1} = s_{t+1}^2 - \hat{h}_{t+1}$$

where m_{t+1} is the forecasting error of the GARCH models and \hat{h}_{t+1} is the forecasted variance which is generated by using the mean and conditional variance equations.

In addition to the traditional mean square error statistics, we use the forecast encompassing framework. The null hypothesis is that model i 's forecast encompasses model j 's. Thus a rejection of the null hypothesis indicates that model j 's forecast contains more relevant information for the forecast. We also run the following three forecast evaluation regressions:

$$(i) \mathbf{s}_{t+s} = a_0 + a_1 \mathbf{s}_{t+s}^{GARCH(1,1)}$$

$$(ii) \mathbf{s}_{t+s} = b_0 + b_1 \mathbf{s}_{t+s}^{alternative\ model}$$

$$(iii) \mathbf{s}_{t+s} = c_0 + c_1 \mathbf{s}_{t+s}^{GARCH(1,1)} + c_3 \mathbf{s}_{t+s}^{alternative\ model}$$

By comparing the R^2 s from these models, we assess forecasting performance of the models relative to the linear t-GARCH (1,1) model.

We report the mean square error statistics for the simple GARCH models in Tables 8 and 9 and for the full model in Tables 10 and 11. For the simple GARCH models the TS-GARCH model does best for the fractionally differenced series. For the first differenced series, the TS_GARCH, as well as the t-GARCH and GJR GARCH perform best. For the full model the non-linear models do well, in particular the EGARCH. Further, the non-linear models outperform the linear models in forecasting the first differenced series.

Tables 12 to 14 report the results of the forecast encompassing test. The second column on the right side of the table records the results of the test that the benchmark tGARCH model encompasses all the information in the alternate models. The third column reports the result for the test that the alternatives contain all the information in the benchmark model. The final column records the result of the tests of whether the combination of both models would lead to an improvement of the forecast generated by either models.

The results from the fractional model in Table 12 indicate that the benchmark models encompass the alternative models and any combination would result in an improvement of the forecast. Unlike the results for the former, the results for 1st difference models, provides evidence on the predictive power of the asymmetric models. The TS-GARCH model performs best over the forecast horizon.

Table 8: Results for 30-day Forecasts for Simple GARCH (1,1) models (Fractional)

	TGARCH	EGARCH	GJR	TS-GARCH	TARCH	APARCH
MSE	0.000144	0.220443	0.000106	0.000000	0.000000	0.000083
RMSE	0.011999	0.469514	0.010289	0.000082	0.000196	0.009108
MAE	0.012010	0.475555	0.010300	0.000063	0.000198	0.009118

Table 9: Results for 30-day Forecasts for Simple GARCH (1,1) models (1st Difference)

	TGARCH	EGARCH	GJR	TS-GARCH	TARCH	APARCH
MSE	0.000000	0.534128	0.000000	0.000000	0.101962	0.000000
RMSE	0.000003	0.730841	0.000003	0.000002	0.319314	0.000003
MAE	0.000003	0.741721	0.000003	0.000002	0.187263	0.000003

Table 10: Results for 30-day Forecast Full Model (Fractional)

	TGARCH	EGARCH	GJR	TS-GARCH	TARCH	APARCH
MSE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
RMSE	0.0003	0.0002	0.0008	0.0010	0.0003	0.0045
MAE	0.0003	0.0002	0.0008	0.0009	0.0003	0.0045

Table 11: Results for 30-day Forecasts Full Model (1st Difference)

	TGARCH	EGARCH	GJR	TS-GARCH	TARCH
MSE	0.0000	0.0000	0.0000	0.0003	0.0000
RMSE	0.0001	0.0000	0.0000	0.0172	0.0000
MAE	0.0000	0.0000	0.0000	0.0131	0.0000

Table 12: Forecast Encompassing Tests Simple GARCH Models (Fractional)

	Forecast Horizon(s)	Ra ²	Rb ²	Rc ²
TGARCH vs. EGARCH	30	0.126	0.080	0.197
TGARCH vs. GJR-GARCH	30	0.126	0.122	0.358
TGARCH vs. TS-GARCH	30	0.126	0.001	0.308
TGARCH vs. APARCH	30	0.126	0.048	0.453
TGARCH vs. TARCH	30	0.126	0.009	0.348

Table 13: Forecast Encompassing Tests Simple GARCH Models (1st Difference)

	Forecast Horizon(s)	Ra ²	Rb ²	Rc ²
TGARCH vs. EGARCH	30	0.154	0.012	0.163
TGARCH vs. GJR-GARCH	30	0.154	0.015	0.154
TGARCH vs. TS-GARCH	30	0.154	0.942	0.944
TGARCH vs. APARCH	30	0.154	0.162	0.167
TGARCH vs. TARCH	30	0.154	0.161	0.191

Table 14: Forecast Encompassing Tests Full GARCH Models (Fractional)

	Forecast Horizon(s)	Ra ²	Rb ²	Rc ²
TGARCH vs. EGARCH	30	0.026	0.070	0.199
TGARCH vs. GJR-GARCH	30	0.026	0.001	0.030
TGARCH vs. TS-GARCH	30	0.026	0.030	0.050
TGARCH vs. APARCH	30	0.026	0.000	0.110
TGARCH vs. TARCH	30	0.026	0.005	0.058

Table 15: Forecast Encompassing Tests Full Models (1st Difference)

	Forecast Horizon(s)	Ra ²	Rb ²	Rc ²
TGARCH vs. EGARCH	30	0.000	0.017	0.017
TGARCH vs. GJR-GARCH	30	0.000	0.002	0.003
TGARCH vs. TS-GARCH	30	0.000	0.942	0.942
TGARCH vs. APARCH	30	0.000	0.013	0.013
TGARCH vs. TARCH	30	0.000	0.030	0.034

Notes: The first column records standard errors for the TGARCH model which represent the null hypothesis that the TGARCH model encompasses the alternative. The second column test whether the alternative model encompasses the TGARCH model. The third column reports the coefficient of determination for the TGARCH model, the fourth column for the alternative model and the final column the combination of the two models.

6. CONCLUSION

The empirical distribution of the exchange rate reflects or validates the tendency of agents to hold relatively long positions in foreign exchange. The key indicators of future market conditions are Jamaica dollar liquidity conditions, spread and the volume of trade.

While the theoretical relation between volume and volatility (i.e. the mixture distribution hypothesis) does not hold in Jamaica, the expected rate of depreciation one day ahead, influences current volatility. An increase in supply to the market does lower the rate of movement in the exchange rate. Expected liquidity conditions one day ahead were found to be important for the mean level of returns but not for volatility. Further current liquidity conditions, proxied by the change in the monetary base, do not seem to be important for market volatility. There is evidence of a DOW effect, where the returns on Mondays are generally higher. This suggests that caution has to be exercised when reacting to movements on a Monday as they may be simply reflecting information generated from the weekend.

There is evidence of a long memory process, which means that shocks to the exchange rate persists for a long period. Further, consistent with expectation, there are spill over effects from the money market and international currency markets. This supports the monetary authority's emphasis on ensuring stable conditions in the financial markets and indicates that policy has to respond quickly to shocks to the foreign exchange market. This is further reinforced by the results that the market response to shocks is asymmetric, reflecting the volatility feedback hypothesis.

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APPENDIX: In-Sample Results for the Simple GARCH Models

Table4: Parameter Estimates for Simple GARCH (1,1) Models (Fractional)

Variables	EGARCH	TGARCH	APARCH	GJR- GARCH	TS- GARCH	TARCH	NGARCH
Mean							
Constant (m)	0.00 (65.84)	0.24 (66.07)	0.24 (1915.77)	0.24 (66.47)	0.37 (84.22)	0.32 (58.62)	0.47 (6.48)
Variance							
Constant (v)	-0.67 (-5.52)	0.00 (0.24)	0.00 (-2.98)	0.00 (0.56)	0.00 (-16.59)	0.00 (-5.71)	0.00 (0.00)
GARCH (beta 1)	0.95 (106.42)	0.77 (41.59)	0.95 (127.63)	0.82 (64.22)	0.00 (15.36)	0.00 (20.52)	0.00 (0.00)
ARCH (Alpha 1)	0.25 (6.78)	0.25 (9.42)	0.00 (3.12)	-0.09 (-1.58)	0.00 (28.03)	0.00 (10.41)	0.00 (0.00)
Gamma1	0.14 (5.51)		-0.48 (-5.74)	0.15 (5.14)		-0.90 (-17.59)	
Delta			0.33 (11.89)				0.00 (0.00)
AIC	2696.02	2694.06	2698.06	2696.060	2694.060	2696.060	2696.060
SBC	2720.77	2713.82	2727.71	2720.760	2713.830	2720.770	2720.760
No. Observations	1034.00	1034.00	1034.00	1034.00	1034.00	1034.00	1034.00
Function Value	5795.37	5447.18	5749.37	5790.01	5504.45	5606.71	4911.63

Table 5: Parameter Estimates for Simple GARCH (1,1) Models (1st Difference)

Variables	EGARCH	TGARCH	APARCH	GJR- GARCH	TS- GARCH	TARCH	NGARCH
Mean							
Constant (m)	0.00 (2.69)	0.00 (2.33)	0.00 (21.17)	0.00 (2.47)	0.00 (4.92)	0.00 (22.93)	0.00 (4.46)
Variance							
Constant (v)	-0.31 (-4.09)	0.00 (0.21)	0.00 (-1.08)	0.00 (0.22)	0.00 (-24.82)	0.00 (-23.28)	0.00 (0.003)
GARCH (beta 1)	0.98 (170.19)	0.79 (54.73)	0.96 (163.16)	0.79 (54.67)	0.00 (34.50)	0.00 (36.76)	0.00 (-0.002)
ARCH (Alpha 1)	0.42 (12.85)	0.25 (11.20)	0.00 (1.11)	0.17 (2.54)	0.00 (16.11)	0.00 (18.68)	0.00 (0.003)
Gamma1	0.06 (3.21)		-0.71 (-9.09)	0.05 (1.39)		-0.42 (-8.21)	
Delta			0.45 (6.29)				-0.02 (-0.002)
AIC	-5784.05	-5786.04	-5782.05	-5784.05	-5786.04	-5784.05	-5784.05
SBC	-5759.34	-5766.28	-5752.40	-5759.30	-5766.28	-5759.34	-5759.34
No. Observations	1034.00	1034.00	1034.00	1034.00	1034.00	1034.00	1034.00
Function Value	6300.75	5945.50	6266.36	6298.74	6172.57	6181.16	5985.44